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HEAT TRANSFER IN A SUPERSONIC FLOW

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Abstract—At present it is commonly assumed that some peculiar surface exists which bounds a viscous region around a body in a supersonic flow. This region is identified with a laminar or turbulent Prandtl layer, the process of transition of visible motion into heat being described by the procedures developed by Prandtl and von Kármán for subsonic flows.

In the present work another approach is used based on the ideas of Osborne Reynolds and the so-called resolution equation where transition of thermal motion into pulsating one is fixed which, in its turn establishes relationship between the Nusselt and Reynolds numbers. This is a power-law relationship in which the coefficients and power exponent may be pre-calculated based on simple considerations. This has been done in the present work.

NOMENCLATURE

- P_{ij}, stress tensor component;
- η, viscosity;
- p, pressure;
- τ, relaxation time;
- l, free molecular path length;
- c, thermal velocity;
- E, total energy of medium;
- $\bar{E}$, relative motion energy;
- Nu, Nusselt number;
- E', energy of mean molar motion;
- Re, Reynolds number;
- M Mach number;
- g, sound velocity;
- γ, adiabatic exponent.

1. TRANSFORMATION OF VISIBLE (MOLAR) MOTION INTO HEAT

It is known from the experiment that the momentum density of material systems does not depend on the amount of heat. This means that thermal motion of the system may be considered relative to that mean motion which is determined for physically visible volumes. All real motions of liquids and gases are always subjected to disturbances due to thermal motions which lead to existence of mean motion of medium with respect to which thermal motion may be considered relative. In this case the fluctuation period of such thermal motion is always short as compared to the period of deviation from the mean motion of medium if the latter is considered to be continuum. If this condition were not obeyed, the equality

$$\Phi = p_{11}e_{11} + p_{12}e_{12} + p_{13}e_{13} + p_{21}e_{21} + p_{22}e_{22} + p_{23}e_{23} + p_{31}e_{31} + p_{32}e_{32} + p_{33}e_{33} = -\rho\frac{\mathrm{d}F}{\mathrm{d}t} \qquad (1.1)$$

would not have been realized within a high degree of accuracy. Here p_{ij} are components of the stress tensor, e_{ij} are components of shear tensor. In this equality the right-hand part is the rate of heat transformation into energy of mean motion.

The reason for isolation of continual and thermal motions which is responsible for the difference in the orders of fluctuation periods of these motions, leads to another phenomenon, i.e. transformation of energy of continual motion with great fluctuation period into the energy of thermal motion with shorter periods of fluctuations.

Reynolds calls such a transition of continual motion into thermal one "transformation" in the true sense. "All similar transformation", Reynolds says, "should come to some definite modifications of true real velocities of matter, depend on the properties of material bodies and be explained by mechanical laws. Therefore, direct transition of the energy of relative continual motion into energy of thermal motion without intermediate stages indicates some reason that changes true velocities of real motion of matter which is exhibited in the form of such a transformation" [1].

A question arises i.e. what should be the form of the phenomenon which Reynolds calls the reason of transformation? Consider the motion of viscous gas. It is known that viscosity may be defined by the product of pressure and relaxation period of the statistical system

$$\eta = p\tau.$$

The time between two collisions of the system may be defined as

$$l = c\tau_0.$$

Here l is the free molecular path length, and c is thermal velocity.

Divide the first relationship by the second

$$\frac{\eta}{l} = \frac{P}{c}\frac{\tau}{\tau_0}.$$

Regarding for the explicit formula for the gas viscosity

$$\eta = \rho c l \varphi$$

we obtain

$$\frac{\varphi\rho c^2}{p} = \frac{\tau}{\tau_0}.$$

Here φ stands for some number depending on the nature of averagings.

Assume that g is squared sound velocity, then

$$\gamma\varphi\frac{c^2}{g^2} = \frac{\tau}{\tau_0}.$$

Since at normal conditions the latter ratio is very small, in this case the transition of visible motion into heat will be unnoticeable.

Nevertheless, motion of the matter of continuum may be of three kinds:

(1) mean observable motion;
(2) disturbed relative, escaping observation motion;
(3) thermal motion that can be observed.

In a bounded medium the wall may presumably intensify largely intermediate motion and even make it visible. This assertion is *a priori*. However, Reynolds' experiments with traced jets confirm it.

Obviously, under certain conditions due to some reason, the energy of mean molar motion with infinite periods is directly transformed into the energy of relative motion with finite periods, this relative motion being characterized by turbulent tortuous motion of the liquid. Relative motion is maintained by continuous transition from the energy of mean motion, despite simultaneous permanent decrease in the energy of relative motion due to transformation into heat [1].

Laminar flows, therefore, differ from potential inviscid flows in that they are disturbed and always include relative irregular flows which transform a laminar flow into a thermal one. Only excitation of these motions may disturb the laminar flow and transform it into turbulent mode.

Thus, according to the above concept the solution of the problem on estimation of the amount of heat obtained due to transition of visible motion into thermal one, fully turns on the methods of calculating the change in the dissipative function within the path equal to the mean wavelength of fluctuations. This calculation was first performed by Reynolds who thus prompted the way of solving heat transfer problems.

2. THE REYNOLDS RESOLUTION EQUATION

If E denotes the total energy of the medium, $\bar{E}$ is the energy of mean molar motion, then under the developed flow conditions the energy of relative motion E' is determined by the formula

$$E' = E - \bar{E}.$$

Following Reynolds we expand the absolute velocity components into mean molar and fluctuational velocities

$$w_1 = \bar{u}_1 + u'_1 \quad w_2 = \bar{u}_2 + u'_2 \quad w_3 = \bar{u}_3 + u'_3.$$

Subscripts 1, 2, 3 show coordinate axes. Components of fluctuational velocity satisfy continuity equation

$$\operatorname{div} \vec{u}' = \frac{\partial u'_1}{\partial x_1} + \frac{\partial u'_2}{\partial x_2} + \frac{\partial u'_3}{\partial x_3} = 0. \qquad (2.1)$$

Assume that the axis x_1 is directed along the velocity of the mean molar flow. Then

$$w_1 = \bar{u} + u'_1; \qquad w_2 = u'_2; \qquad w_3 = u'_3.$$

In this case the net force of the absolute motion may be expressed as

$$E = \bar{E} + E' + \rho \bar{u} u'_1 = E + E' + \xi.$$

The addition of energy ξ determines transition of visible motion into heat, and the rate of its time change may be presented as

$$\frac{d\xi}{dt} = (\vec{\bar{u}}, \operatorname{grad} \xi).$$

It may be shown that one portion of the rate of energy change ξ is used to overcome the inertia forces, the other is transformed into heat. The latter may be expressed by the following mathematical expression

$$\frac{d\xi_m}{dt} = \rho u_1'^2 \frac{\partial \bar{u}}{\partial x_1} + \rho u'_1 u'_2 \frac{\partial \bar{u}}{\partial x_2} + \rho u'_1 u'_3 \frac{\partial \bar{u}}{\partial x_3}. \qquad (2.2)$$

Reynolds considers the case when the velocity $\bar{u}$ slightly changes along the stream line and the flow possesses axial symmetry. In this case the component u'_3 may be neglected, and equation (2.2) takes the form

$$\frac{d\xi}{dt} = \rho u'_1 u'_2 \frac{\partial \bar{u}}{\partial x_2}.$$

By including solutions of equation (2.1) into this equality and transforming the Rayleigh dissipative function based on the same solutions, and comparing the results, Reynolds has obtained his outstanding equality which allows estimation of transition of visible motion into heat. This equality may serve as a basis of the theory of heat transfer of bodies interacting with a hydrodynamic flow.

Then Reynolds affirms that, in a wall region, integrals of unsteady differential equations for viscous fluid cannot exist under any conditions, since they are not consistent with the medium at the intial moment. This statement is grounded by Reynolds' paper [2]. Therefore, for the wall region the viscous fluid equations should have the form

$$\operatorname{grad} p = \eta \Delta_2 u.$$

Reynolds seeks for a peculiar solution for this differential equation, and expresses it in terms of the mean velocity over the thickness of the region, i.e. in terms of u_m which is equal to

$$u_m = \frac{1}{\delta} \int_0^\delta u \, dx_2.$$

By including these solutions into his basic equality which governs the transition of visible motion into a thermal one, Reynolds reduces it to the form

$$\rho \frac{u_m \delta}{\eta} = \frac{2\delta^3}{3} \frac{\Phi}{d\xi_m/dt}. \qquad (2.3)$$

This is the final form of the Reynolds resolution equation.

After introduction of the following notation

$$\rho \frac{u_m \delta}{\eta} = Re_\delta \qquad l = \frac{2\omega\delta}{\pi}$$

equation (2.3) takes the form

$$\tfrac{3}{2} Re_\delta = \left(\frac{\pi}{2}\right) \frac{l^4 + 2 \times 5{\cdot}53\, l^2 + 50}{0{\cdot}95\, l} \quad (2.4)$$

or

$$0{\cdot}247\, Re_\delta = \frac{l^4 + 11{\cdot}06\, l^2 + 50}{l}. \quad (2.5)$$

Assume the following relationship between the numbers Nu, Re and period of spatial fluctuation

$$Nu_L = f(Re_L)\,\varphi(\psi) = f(Re_L)\,\psi\left(\frac{l}{\delta}\right).$$

With account for

$$Re_L = Re_\delta \frac{\delta}{L},$$

obtain

$$Nu_L = f(Re_L)\,\psi\left(\frac{l Re_\delta}{Re_L L}\right).$$

Now we make use of the Reynolds resolution equation (2.5). It may be approximated by the power-law function, i.e. it may be assumed that

$$Re_\delta = B^m l^m.$$

Substitution of this relationship into the latter equality gives

$$Nu_L = f(Re_L)\,\psi\left(\frac{l^{m+1} B^m}{Re_L L}\right).$$

After separation of the function depending only on Re_L from this expression, we shall have

$$Nu_L = \psi(Re_L)\,\chi\left(\frac{l^{m+1} B^m}{L}\right).$$

The way the heat-transfer phenomena proceed makes such a transition possible. In almost all of the cases the experiments allow the number Nu to be expressed in the form of the power-law function of Re_L, the factor and power exponent constant. Therefore the function

$$\chi = \left(\frac{l^{m+1} B^m}{L}\right)$$

may also be assumed constant. The numerical value of this function may be found from the analysis of the phenomena at any convenient point. The stagnation point of the incident flow may be the case.

3. HEAT FLUX FROM A SUPERSONIC GAS FLOW TO A SOLID

Unlike a subsonic flow, a supersonic one around a solid is divided into three regions. The first region corresponds to the state of fluid from the infinite distance to the shock wave: the second, to the state of fluid from the shock wave to the surface around the solid in the flow with discontinuous change of the density, and at last the third one corresponds to the hydrodynamic state of the fluid between this surface and that of the solid body. The picture quite agrees with observations. We think the transition occurs in the third region. This does not contradict the generally accepted point of view.

The third region is acknowledged by all investigators and up to now is identified with the Prandtl–Kármán boundary layer. As the majority of investigators think, this region, as well as in a subsonic flow, is divided into laminar and turbülent ones, as far as the flow pattern is considered.

We do not hold to the view of the majority.

Consider the region of transition of visible motion into heat for a supersonic flow around solids to be bounded from one side with the solid surface, and from the other, with the surface not far from that of the solid body where the velocity undergoes discontinuous change. Then assume that all the values which determine the state of the medium in the region of transition of visible motion into heat, depend on the coordinates and time only through the variable $\xi = V + gt$. Thus we ascertain that transition of visible motion into heat proceeds in the form of a front. It may be stated *a priori* that such a process is possible, but this means that it is sure to exist. This alternative may be theoretically resolved only by way of experiments and we shall appeal it to.

Earlier the formula

$$g\tau\varphi = \rho g(u_0 - u) - k\frac{\mathrm{d}T}{\mathrm{d}n}$$

was established. The right-hand part of the equation is evidently that energy heat flux which is absorbed by the solid body.

This heat flux is known to be determined in terms of the heat-transfer coefficient as

$$\alpha(T = T_0).$$

With regard for the above, have

$$\alpha(T - T_0) = g\tau . \Phi. \tag{3.1}$$

The product $g\tau$ is related to the conditions of flow behaviour and does not depend on temperature. Differentiation of the relationship (3.1) with respect to temperature yields

$$\alpha = g\tau\frac{\mathrm{d}\Phi}{\mathrm{d}T}. \tag{3.2}$$

Thus, a simple relationship is established between the heat-transfer coefficient and the Rayleigh function. This is very significant since Reynolds has developed a method of calculating the energy of transition of visible motion into heat using the Rayleigh function. Now we shall try to make use of the method. g_u is the phase velocity of fluctuations, λ is the wave-length of fluctuations. Then we have

$$\lambda = g_u\tau.$$

But Reynolds expressed his resolution equation through the variable l

$$l = \frac{2\omega\delta}{\pi} = \frac{4\delta}{\lambda}. \tag{3.3}$$

Account should be taken of the fact that the spatial period ω is equal to $2\pi/\lambda$. From comparison of the relationships given, it follows that

$$\frac{4\delta}{l} = g_u\tau; \tau = \frac{4\delta}{g_u l}.$$

Using the latter relationship we exclude the period τ from formula (3.2) and arrive at

$$\alpha = \frac{4g\delta}{g_u l}\frac{\mathrm{d}\Phi}{\mathrm{d}T}. \tag{3.4}$$

The value δ is the measure of the thickness of the zone where visible motion is transformed into heat. Introduction of the time of zone formation τ makes evident the identity

$$\delta/g = \tau'.$$

Exclusion of the value δ from formula (3.4) yields

$$\alpha = \frac{4g^2\tau'}{g_u l}\cdot\frac{\mathrm{d}\Phi}{\mathrm{d}T}. \tag{3.5}$$

Express this relationship in terms of the numbers Nu and Re.

The line obtained when the solid body is sectioned by a plane through the abscissa is assumed to be the measure of the length. The point of flow incidence is considered to be a reference point of the line and its end is the point where the local heat-transfer coefficient is sought. Besides, we use the known Maxwell relationship

$$k = fc_v\eta.$$

The mean value of the shear flow velocity in the region of transition of visible motion into heat is W_t.

Now we shall divide equality (3.5) by the Maxwell relationship and multiply it by the line length L. We arrive at

$$Nu_L = \frac{\alpha L}{k}; \qquad Re_L = \frac{W_t\rho L}{\eta}$$

$$Nu_L = Re_L\cdot\frac{4g^2}{fg_u W_t l}\cdot\frac{\mathrm{d}\Phi}{\mathrm{d}T}\cdot\frac{\tau'}{c_v\rho}.$$

But the product $\tau . \mathrm{d}\Phi$ is modification of the heat amount due to transition of visible motion into heat. The product $c_v\rho\,\mathrm{d}T$ expresses the same. Their ratio should therefore, be considered equal to unity. This condition allows the last formula to be written as follows

$$Nu_L = Re_L\frac{4g^2}{fg_u W_t l}. \tag{3.6}$$

Now it is of the form which allows its transformation using the Reynolds resolution equation.

The Reynolds resolution equation is of the form

$$0{\cdot}247\, Re_\delta = \frac{l^4 + 11{\cdot}06\, l^2 + 50}{l}. \qquad (3.7)$$

Within any range of two variables Re_δ and l, may be approximated by the power-law formula that

$$Re_\delta = B^m l^m,$$

where B^m is a constant. Also, the identity

$$Re_L = Re_\delta \frac{L}{\delta}$$

holds. With regard for these two relationships formula (3.6) may be written as follows

$$Nu_L = Re_L^{(m-1)/m} \cdot \frac{4Bg^2}{fg_u W_t}\left(\frac{L}{\delta}\right)^{(1/m)}$$

or

$$\left.\begin{aligned} Nu_L &= A\, RE_L^{(m-1)/m} \\ A &= \frac{4Bg^2}{fg_u W_t}\left(\frac{L}{\delta}\right)^{(1/m)}. \end{aligned}\right\} \qquad (3.8)$$

The way of calculation of the constants m and A would be the way of theoretical solution of heat-transfer problems. It appears possible to find it in many particular cases.

If formulae (3.7) hold for any points of surface, they will also hold at the point of flow incidence. Thus, in the cases of clear physical picture at the point of flow incidence, the possibility of calculating heat losses for any point of the heat transfer surface arises simultaneously.

The stagnation point of the incident flow cannot be considered a mathematical one. Therefore, at this point, the length L has some limit value L_0. The thickness of the region of transition of visible motion into heat also has some limited value δ_0.

As to the three values g, g_s and W_t, the following considerations may be presented. At the stagnation point the front velocity and the phase velocity of fluctuations coincide and are equal to the speed of sound. Thus we affirm that both phenomena are due to the same reason, i.e. fluid compression.

Transition of visible motion into heat at the stagnation point of the incident supersonic flow should occur at the Mach number equal to unity, if the flow velocity is related to the sound speed of the fluid which thermodynamic state corresponds to its state at the stagnation point. This means that all three values are equal.

All this allows the formula for heat transfer in a supersonic flow to be written in the form

$$Nu_L = \frac{4B}{f}\left(\frac{L_0}{\delta_0}\right)^{1/m} . Re_L^{(m-1)/m}. \qquad (3.9)$$

The Reynolds resolution equation (3.7) also allows calculation of the ratio L_0/δ_0 for the process at the stagnation point of the incident flow.

At the stagnation point the value l should have the minimum value which can be neglected as compared to figure 50. Then we shall have

$$0{\cdot}247\, Re_{\delta_0} = \frac{50}{l}. \qquad (3.10)$$

When defining Re and taking account of

$$l = \frac{4\delta_0}{\lambda_0}$$

we have

$$\frac{0{\cdot}247 \,.\, \rho W_t \delta_0}{\eta} = \frac{50}{4} \cdot \frac{\lambda_0}{\delta_0}.$$

But viscosity is described by the relationship

$$\eta = \tfrac{1}{3}\rho c s.$$

where c is thermal velocity, s is free path length.

Now the Reynolds resolution equation may be written as

$$\frac{0{\cdot}741\, W_t \delta_0}{cs} = \frac{25}{2}\,\frac{\lambda_0}{\delta_0}.$$

Or, when substituting the thermal velocity by the speed of sound

$$\frac{0.741\sqrt{\gamma}}{\sqrt{3}}\frac{\delta_0}{s} = \frac{25}{2}\cdot\frac{\lambda_0}{\delta_0}.$$

Hence it follows

$$\frac{\delta_0^2}{s\lambda_0} = \frac{25}{1.482} \times \sqrt{\left(\frac{3}{\gamma}\right)}. \quad (3.11)$$

One molecule, on the average, occupies the volume

$$\frac{4}{3}\frac{\pi s^3}{8} = \frac{\pi s^3}{6}. \quad (3.12)$$

The layer with transformation of visible energy into heat is of thickness δ_0. Hence, the minimum volume of this layer including one molecule will be $\delta_0 s^2$. This volume should be equivalent to the half volume per one molecule in free space. Thus.

$$\delta_0 s^2 = \frac{\pi}{12}s^2.$$

Using this relationship the free path length can easily be excluded from formula (3.12)

$$\frac{\delta_0}{\lambda_0} = \frac{25 \times 12}{1.482\pi} \times \sqrt{\left(\frac{3}{\gamma}\right)}.$$

In order to calculate the unknown ratio, the relationship between the minimum value of L_0 and the wavelength should be known.

If the maximum value of the fluctuation amplitude coincides with the geometric point of the stagnation region, then, obviously, the minimum value of the magnitude L should be equal to one fourth of the wavelength. This condition allows final expression for the ratio L_0/δ_0 in the form

$$\frac{L_0}{\delta_0} = \frac{\pi \times 0.0148}{12}\sqrt{\left(\frac{\gamma}{3}\right)} = 0.00389\sqrt{\left(\frac{\gamma}{3}\right)}.$$

With polyatomic gases, for which γ may be assumed equal to 1·44, we shall have

$$\frac{L_0}{\delta_0} = 0.0027.$$

Thus for polyatomic gases heat-transfer formula (3.9) may be rewritten as

$$Nu_L = \frac{4B}{f}(0.0027)^{1/m} Re_L^{(m-1)/m} = B' Re^{(m-1)/m}. \quad (3.13)$$

The proposed method of calculation of the heat transfer coefficient using the Reynolds resolution equation would be rather complete if the constants m and B might be calculated by the main experimental data. Such a calculation appears to be possible. This will be done when considering particular examples.

4. SOME EXAMPLES OF CALCULATING HEAT TRANSFER OF SOLIDS IN A GAS-DYNAMIC FLOW

To solve any heat-transfer problem for a gas-dynamic flow the approximation region of the power-law function of the Reynolds resolution equation should be defined. In other words, the values of m and B should be estimated by the main parameters for any particular case.

The Mach number and thermodynamic state of fluid in the premix chamber of a gas-dynamic nozzle may be assumed to be the main parameters of the gas-dynamic flow. Also, the nature of the fluid should be taken into account.

We shall write the Mach number for the stagnation point. It is this number that characterizes the gas-dynamic flow. The sound speed of the fluid at the initial state is g_s. Then

$$M = \frac{W}{g_s}.$$

By multiplying the nominator and denominator by the time of formation of the region of transition of visible motion into heat obtain

$$M = \frac{W\tau'}{g_s\tau'}.$$

Since for the stagnation point

$$W = W_t = g,$$

the latter identity may be rewritten as

$$M = \frac{\delta}{g_s \tau'}.$$

Now we shall multiply the nominator and denominator of the identity by the fluctuation wavelength and obtain

$$M = \frac{\delta_0 \lambda_0}{g_s \tau' \lambda_0}.$$

According to Reynolds, the ratio $4\delta_0/\lambda_0$ is equal to the limit number. Therefore

$$4M = \frac{l_0 \lambda_0}{g_s \tau'}.$$

Substitute in this expression the sound speed by thermal velocity using formula

$$g_s = \sqrt{\left(\frac{\gamma}{3}\right)} c_M$$

where c_M is the thermal velocity of a molecule at the initial state. Then for the Mach number obtain

$$4M = \sqrt{\left(\frac{3}{\gamma}\right)} \frac{l\lambda_0}{c_M \tau'}. \quad (4.1)$$

It is seen from the relationship that in order to estimate the lower boundary of the value l variation, the ratio

$$\frac{c_M \tau'}{\lambda_0}$$

should be known. This ratio can be estimated when Reynolds' ideas are successively used.

According to Reynolds, thermal motion of molecules is relative as compared to the macroscopic motion of medium. The fluctuational motion which through the macroscopic motion of medium is transformed into heat is, in its turn, relative to the mean macroscopic motion. This statement gives us a right to consider that the mean kinetic energy of the molecules of a medium at the final state is equal to the sum of the kinetic energy of molecules at the initial state of the medium and the kinetic energy of the fluctuational motion transformed into heat. Mathematically this can be written as

$$\rho_M c_M^2 + \rho_M u^2 = \rho c^2.$$

Denoting the final to initial density ratio by β, obtain

$$c_M^2 + u^2 = \beta c^2. \quad (4.2)$$

This expression is equivalent to the vector equality

$$\mathbf{c}_M + \mathbf{u} = (\sqrt{\beta})\mathbf{c}.$$

Thus, any translational thermal motion of molecules transits into another translational motion along the vector $\mathbf{c}$ due to fluctuations.

Multiply equation (4.2) by the squared time of formation of the region of visible motion transition into heat at the stagnation point

$$(c_M \tau')^2 + (u\tau')^2 = \beta (c\tau')^2.$$

Substitution of c by the sound speed g yields

$$(c_M \tau')^2 + (u\tau')^2 = \beta \left[\sqrt{\left(\frac{3}{\gamma}\right)} g\tau'\right]^2 = \frac{3\beta}{\gamma} \delta_0^2. \quad (4.3)$$

For the time of formation of the transition zone a fluctuating particle should cover the distance equal to the wavelength and return to the initial state. Under this condition only, fluctuational motion is reasonable to be considered relative with respect to the mean motion which is relative to the thermal one. This means that the product $u\tau'$ is equal to the double fluctuation wavelength, i.e.

$$u\tau' = 2\lambda_0.$$

At this condition equality (4.3) may be written as

$$\left(\frac{c_M \tau'}{\lambda_0}\right)^2 + 4 = \frac{3\beta}{\gamma} \frac{\delta_0^2}{\lambda_0^2} = \frac{3\beta}{16\gamma} l^2.$$

Hence follows

$$\left(\frac{\lambda_0}{c_M \tau'}\right)^2 = \frac{16\gamma}{(3\beta l^2 - 64\gamma)}. \quad (4.4)$$

When squared and including the expression obtained, equality (4.1) takes the form

$$M^2 = \frac{3l^2}{3\beta l^2 - 64\gamma}.$$

Then

$$l = 8\sqrt{\left(\frac{\gamma}{3}\right)}\frac{M}{\sqrt{(\beta u^2 - 1)}}. \qquad (4.5)$$

Thus, the formula derived allows estimation of the lower boundary of the Reynolds variable using main parameters M, β and γ.

At the Power Engineering Institute O. N. Kastelin and L. N. Bronsky [3] have studied heat transfer in a supersonic flow of spherical and elliptical bodies. Their experiments may be used to verify the above considerations.

These authors used spherical bodies 1·5 and 2 cm i.d. Elliptical bodies were of two types. One group had a large half-axle of 1·5 and 2 cm and a smaller half-axle of 0·75 and 1 cm. The other group had a large half-axle of 0·75 and 1 cm and a smaller one of 0·375 and 0·5 cm.

The first group of elliptical bodies was arranged with their large half-axle against the incident flow. The second group had their smaller half-axle against the incident flow. The tests have been carried out on a gas-dynamic installation of continuous operation at M = 2·77, 2·40 and 1·88. The temperature behind the shock wave was 274, 270 and 261°C.

From the experiments O. N. Kastelin and L. N. Bronsky have found the relationship for all of the above bodies

$$Nu_L = 0{\cdot}8_L^{0{\cdot}5}.$$

Here the numbers Nu_L and Re_L were defined in the same way as in the previous section. The factor in the Reynolds number was estimated as the mean of some observations. It ranged from 0·7 to 0·9.

If the Mach numbers presented by Kastelin and Bronsky are used, then in accordance with (4.5) we may tabulate the lowest values of the variable l in the Reynolds resolution equation. For the table calculations the gas was assumed polyatomic, and, therefore, γ equal to 1·44 and β equal to unity.

M	l
2·77	6·00
2·40	6·13
1·88	6·55

The table shows that the minimum value of the variable is about 6. The value of l increases with the distance from the stagnation point

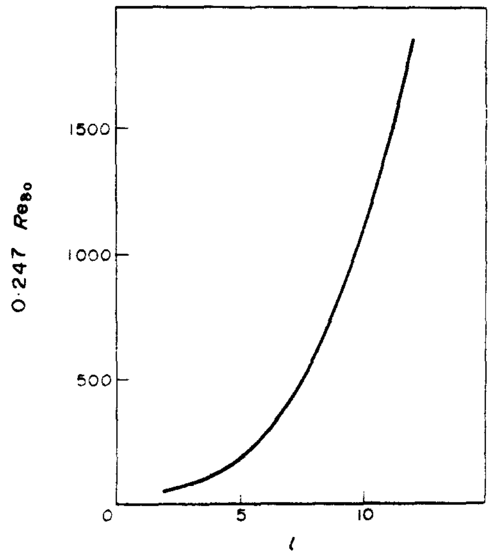

FIG. 1. Graphic presentation of Reynolds resolution equation.

(Fig. 1). Approximation of the resolution equation in this region by the power-law function yields

$$Re_\delta = 42l^2.$$

The general form of the approximating function is

$$Re = B^m l^m.$$

Comparison of these two formulae gives the following values of B and m

$$m = 2; \qquad B^2 = 42; \qquad B = 6{\cdot}50.$$

The data obtained allow formula (3.13) to be written for the given case as

$$Nu = \frac{4 \times 6{\cdot}50}{f} 0{\cdot}0027^{\frac{1}{2}} . Re^{0{\cdot}5}. \qquad (4.6)$$

From the theoretical predictions by Eucken for polyatomic gases, the Maxwell factor is 1·8. Substitution of this value for f in formula (4.6) yields

$$Nu = 0{\cdot}75\, Re^{0{\cdot}5} \qquad (4.7)$$

Thus, the theoretical formula for heat transfer of solid s in a gas-dynamic flow is fully confirmed by experimental data.

The question arises how far it may be extrapolated with increasing Mach numbers.

For very large Mach numbers formula (4.5) can approximately be written as

$$l = 8\sqrt{\left(\frac{\gamma}{3\beta}\right)}.$$

But for the density ratio β according to the Clapeyron equation the expression

$$\beta = \frac{\rho}{\rho_0} = \frac{pT_0}{p_0 T}$$

is valid.

In accordance with (4.2) pressures should be assumed equal. Then we have

$$\beta = \frac{T_0}{T}.$$

Let the temperature corresponding to the initial state of the substance be equal to 300°K, and that to the final state, to 9000°K. Then

$$\beta = \frac{300}{9000} = \frac{1}{30}.$$

If for polyatomic gas $\gamma = 1{\cdot}44$, then in the resolution equation

$$l = 8 \times 1{\cdot}2 \times (\sqrt{10}) = 9{\cdot}6 \times 3{\cdot}16 = 30{\cdot}4.$$

With these values of l the resolution equation is

$$Re_\delta = \frac{l^3 + 11{\cdot}06\, l}{0{\cdot}247}.$$

For the range of l from 30 to 40 the above quality may be approximated by the formula

$$Re_\delta = 135\, l^2.$$

Hence we have

$$m = 2 \qquad B = (\sqrt{135}) = 11{\cdot}6.$$

The heat-transfer formula, therefore, will be of the form

$$Nu_L = \frac{4 \times 11{\cdot}6 \times 0{\cdot}05}{1{\cdot}8} Re_L^{0{\cdot}5} = 1{\cdot}35\, Re^{0{\cdot}5}.$$

It is seen from the formula that at M corresponding to stagnation temperature of 9000°K the heat-transfer coefficient increases by about two times as compared to the experimental data of O. N. Kastelin and L. N. Bronsky.

O. N. Kastelin and L. N. Bronksy have also investigated the heat transfer of a plate normal to the incoming flow direction. They have obtained the formula

$$Nu_L = 0{\cdot}31\, Re_L^{0{\cdot}59}. \qquad (4.8)$$

It may be shown, however, that the latter formula corresponds to the one obtained earlier.

In fact, assume that in the Reynolds resolution equation the lower boundary of the variable l in the earlier experiments corresponds to that in the latest experiments. Then the difference between formulae (4.8) and (4.7) should be explained only by the method of approximation.

When deriving formula (4.7) the Reynolds resolution equation was approximated by the power-law function

$$B^m l^m = 42\, l^2; \qquad m = 2; \qquad B = (\sqrt{42}) = 6.5.$$

The same formula may also be approximated by the power-law function of the form

$$B^{2{\cdot}5}\, l^{2{\cdot}5} = 42\, l^2; \qquad m = 2{\cdot}5.$$

In this case the ratio $(m - 1)/m$ will be equal to

0·6, which almost corresponds to the power exponent of the second formula by O. N. Kastelin and L. N. Bronsky. We have to calculate only the constant B_1. Assume, as before, that the lower boundary of the value l is 6. Then we shall have

$$B^{2\cdot 5} = \frac{42}{6^{0\cdot 5}} = 17\cdot 1.$$

Hence

$$B_1 = 3\cdot 11.$$

Now the heat-transfer formula may be presented in the form

$$Nu_L = \frac{4 \times 3\cdot 11 \times 0\cdot 052}{1\cdot 8} Re_L^{0\cdot 6} = 0\cdot 361\, Re^{0\cdot 6}.$$

As we see, the formula is found which is similar to (4.7). Therefore, there are no reasons to think that heat transfer in the latest experiments of O. N. Kastelin and L. N. Bronsky differs from that in their earlier experiments.

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TRANSFERT DE CHALEUR DANS UN ECOULEMENT SUPERSONIQUE

Résumé—On suppose actuellement qu'il existe une surface particulière qui délimite une région visqueuse autour d'un corps dans un écoulement supersonique. Cette région est identifiée à une couche laminaire ou turbulente de Prandtl, le processus de transition du mouvement visible vers la chaleur étant décrit par les méthodes développées par Prandtl et Karman pour des écoulements subsoniques.

Dans la présente étude, on a utilisé une autre approche basée sur les idées d'Osborne Reynolds et cette équation de résolution portant son nom où est fixée la transition d'un mouvement thermique à un autre mouvement pulsatoire qui, à son tour, établit une relation entre les nombres de Nusselt et Reynolds. C'est une relation en loi de puissance dans laquelle les coefficients et les exposants de puissance peuvent être précalculés à partir de considérations simples, comme cela est montré dans cet article.

WÄRMEÜBERGANG IN ÜBERSCHALLSTRÖMUNGEN

Zusammenfassung—Gegenwärtig wird allgemein angenommen, dass eine besondere Oberfläche existiert, die eine Zähigkeitszone um einen Körper in einer Überschallströmung begrenzt. Diese Zähigkeitszone wird als laminare oder turbulente Prandtl'sche Grenzschicht identifiziert, wobei der Übergangsprozess zwischen Bewegung und Wärme durch das Verfahren beschrieben wird, das Prandtl und Kármán für Unterschallströmungen entwickelt haben.

In der vorliegenden Arbeit wird eine andere Näherung benutzt. Ihr liegen die Ideen von Osborne Reynolds und die sogennante Lösungsgleichung zu Grunde, in der ein bestimmter Übergang zwischen Längsbewegung und pulsierender Bewegung festgelegt ist. Daraus ergibt sich ein Zusammenhang zwischen Nusselt-Zahl und Reynolds-Zahl und zwar in Form eines Potenzgesetzes, in dem die Koeffizienten und Exponenten aufgrund einfacher Überlegungen vorausberechnet werden können. Dies wurde in der vorliegenden Arbeit durchgeführt.

О ТЕПЛООБМЕНЕ В СВЕРХЗВУКОВОМ ПОТОКЕ

Аннотация—В настоящее время общепринято предположение о существовании особой поверхности, ограничивающей область вязкого течения около тела, обтекаемого сверхзвуковым потоком. Эту область отождествляют с ламинарным или турбулентным слоем Прандтля и процесс превращения видимого движения в тепло описывают теми приемами, которые разработаны Прандтлем и Карманом для дозвуковых течений.

В настоящей работе используется другой прием, основанный на идеях Осборна Рейнольдса и его так называемом разрешающем уравнении, в котором фиксируется переход тепловых движений в пульсационные, что в свою очередь дает связь между критериями Нуссельта и Рейнольдса. Связь эта имеет степенной вид, причем входящие в нее коэффициент и показатель степени можно предвычислить исходя из довольно простых соображений, что и проделано в данной работе.